# Design of drone reconnaissance route based on Integer Programming

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**Abstract:** When a natural disaster strikes an area, it is very important to design a reasonable drones reconnaissance path in order to quickly grasp the disaster situation.This paper will establish a mathematical model based on Integer Programming, to solve a multi-base and multi-target drone reconnaissance problem.The hurricane in Puerto Rico(2019 MCM Problem B) is taken as an example and the model is applied to find the shortest path to the detection target. Noted that finding the shortest path is to grasp the disaster situation in the shortest time (assuming that the shorter the path, the shorter the detection time), so it will play an important role in disaster relief.

## 1. Introduction

The attack of extreme weathers can cause severe damage to a region such as casualties, Houses collapsed, loss of power, traffic jams. In 2017, the worst hurricane to ever hit the United States territory of Puerto Rico, knocked down 80 percent of Puerto Rico's utility poles and all transmission lines, resulting in loss of power to essentially all of the island's 3.4 million residents. The electrical power and cell service outages lasted for months across much of the island, and longer in some locations. Widespread flooding blocked and damaged many highways and roads across the island, making it nearly impossible for emergency services ground vehicles to plan and navigate their routes.

One Non-governmental organization (NGO) – HELP, Inc. - is attempting to improve its response capabilities by designing a transportable disaster response system called “DroneGo”. DroneGo will use rotor wing drones to deliver pre-packaged medical supplies and provide high-resolution aerial video reconnaissance.

In order to get relief operations going as quickly as possible, NGO need to send drones from bases to monitor the devastation in Puerto Rico. Aiming at the shortest flight distance, the model designs a drone reconnaissance path based on the geographic information of Puerto Rico.

## 2. Problem description

Puerto Rico has three drone bases, each with the same type of drone for reconnaissance missions. The range of a fully charged drone is 112.54km. The geographic location of seven sites that NGO needs to investigate before launching rescue operations and 3 drone bases are shown in the table below:

Table 1: Latitude and Longitude of reconnaissance points and drone bases

| Reconnaissance points | Latitude | Longitude |
|---|---|---|
| 1 | 18.33 | -65.65 |
| 2 | 18.22 | -66.03 |
| 3 | 18.44 | -66.07 |
| 4 | 18.40 | -66.16 |
| 5 | 18.47 | -66.73 |

| | | |
|---|---|---|
| 6 | 18.27 | -66.70 |
| 7 | 18.23 | -66.39 |
| 8 | 18.22 | -66.23 |
| Drone bases | Latitude | Longitude |
| A | 18.40 | -66.52 |
| B | 18.25 | -66.27 |
| C | 18.32 | -65.87 |

Assumption:

- The drone completes the detection of the target point after passing the detection point, regardless of the duration of the drone stay at the target point.
- The problem of the drones slowing down when turning is not considered, the drones always flies at a constant speed.
- Each base only has one drone available(this assumption is reasonable since path planning is needed when there is a shortage of drones).

## 3. Mathematical formulation

The goal of the model is to minimize the time the drone spends on reconnaissance. Since we assume that the drone moves at a constant speed, we only need to make the drone fly the shortest distance and take the time when the last drone returns to the base as the time to complete the mission. The model is as follow:

To clarify the model, we will use the following notations:

- $x_{ij}$ is the 0-1 variable indicating whether there is a path between two points.
- $Y_A$ is the point the drone passes before returning to base A. $Y_B, Y_C$ is similar.
- $S_i$ $(i = A, B, C, 1, 2..., 7)$ is the cumulative distance traveled by the drone at a certain point.
- $S_A^{'}$ is the cumulative flight distance of the drone travel back to base A. $S_B^{'}$ , $S_C^{'}$ is similar.
- $d(i, j)(i, j = A, B, C, 1, 2..., 7)$ is the distance between two points.

Model:

$$\min\ f = \min\{\max(S_A^{'}, S_B^{'}, S_C^{'})\}$$

$$
s.t.\begin{cases}
\sum_{i=1}^{7} x_{ij} = 1 \quad (j = 1,2...,7) & (1)\\
\sum_{j=1}^{7} x_{ij} = 1 \quad (i = 1,2...,7) & (2)\\
\sum_{i\in S}\sum_{j\in S} x_{ij} \le |S| - 1 \quad S \subset \{1,2...,7\} & (3)\\
Y_A = \begin{bmatrix} x_{1A} & x_{2A} & x_{3A} & x_{4A} & x_{5A} & x_{6A} & x_{7A} \end{bmatrix}^T & (4)\\
Y_B = \dots ; Y_C = \dots & \\
S_A^{'} = d(A, Y_A) + \begin{bmatrix} S_1 & 0 & \cdots & 0 \\ 0 & S_2 & & \\ \vdots & & \ddots & \\ & & & \\ 0 & & & S_7 \end{bmatrix} Y_B & (5)\\
S_B^{'} = \dots ; S_C^{'} = \dots & \\
S_j = d\left( j, \sum_{i=1}^{7} x_{ij} i \right) + \sum_{i=1}^{7} x_{ij} S_i & (6)\\
S_A^{'}, S_B^{'}, S_C^{'} \le 112.54 & (7)\\
x_{ij} \in \{0,1\}; \quad S_A, S_B, S_C = 0 & (8)
\end{cases}
$$

To understand the model above, let's explain each constraint:

- Constrains (1) and (2) means that each point has only one arc exit and one arc entry, which ensures the drone only needs to pass each point once.
- Constrain (3) ensures that there is no loop between reconnaissance points.
- Constrain (4): The column vector is used to store the points that the drone passes before returning to base A. For example, the drone passes points 1 and 2 before returning to base A if $x_{1A}, x_{2A}$ equals to 1.
- Constrains (5) and (6) mean that the cumulative distance of each point is equal to the cumulative distance of the previous point that the drone passed plus the distance between these two points.
- Constrain (7) means that the cumulative distance of the drones back to the base should be less than 112.54km.
- Constrain (8) define $x_{ij}$ as a 0-1 variable, and set the cumulative distance at points A, B, and C to be 0km.

## 4. Result and analysis

Using lingo to find the optimal solution, result is shown below: The red, green, blue lines represent the paths of the Drone departing from base A,B,C respectively.

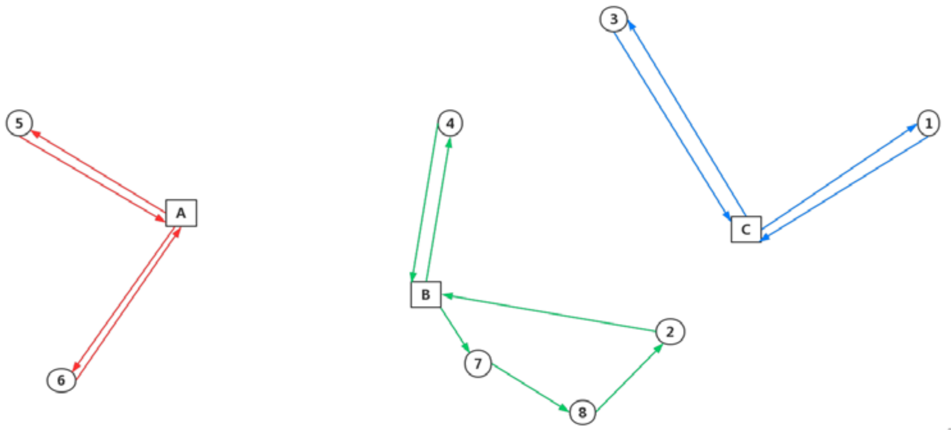

Fig. 1 result of shortest reconnaissance path

The minimum value of the objective function is 105.34, which means the last drone completing the mission has a flight distance of 105.34km. NGO can complete reconnaissance of all locations within 40 minutes.

This problem is similar to the Traveling Salesman Problem(TSP), which is an NP problem when the scale of the problem is large. When the number of reconnaissance points increase to tens or even hundreds, the efficiency of using Integer Programming to solve the problem will become extremely low. One approach is to use intelligent algorithms such as Genetic Algorithms(GA) and Simulated Annealing Algorithms(SAA).However, the design and implementation of intelligent algorithms are often difficult and may only get the local optimal solution.

## 5. Conclusion

This paper provides a solution based on integer programming for path planning, which is quite effective when the scale of the problem is low. Noted that As the scale of the problem increases, the intelligent algorithm is more suitable to solve the problem than the Integer Programming.

The strength is that the model is easy to adjust to different situations, since the objective function and constraints can be adjusted according to the actual situation. But the weakness is also obviously that it is only suitable for solving small-scale problems. Therefore, I will explore the application of intelligent algorithm in this problem in the subsequent research.

## References

[1] Junjae Chae,Amelia C. Regan. A mixed integer programming model for a double row layout problem[J]. Computers & Industrial Engineering,2020,140.

[2] Chuanjun Chen,Xianrong Yu,Yanan wang,Yubo Sun. Dam Location Problem Based on 0 —1 Integer Programming Model[J]. Journal of Yantai University (Natural Science and Engineering Edition), 2018,31(01) :1-5.

[3] Gang Sun, Chunsheng Hao, Shan YU. Routing Inspection Method of Power Cable and Based on Integer Programming[J]. Hebei Electric Power, 2016,35(03):50-52.

[4] Jiqiang Wang. LINGO-based modeling methods for the traveling salesman promblem[J]. Computer Engineering & Science, 2014,36(05):947-950.

[5] Fiedrich, F., Gehbauer, F., Rickers, U., 2000. Optimized resource allocation for emergency response after earthquake disasters. Safety Science 35 (1), 41–57.

[6] Zhan J, Xiewen J, Guo Q, et al. The problem of multi-uav reconnaissance and scheduling under uncertain conditions [J]. Fire and command and control, 2018(10).